Please attempt the following questions in preparation for the online session on 22nd February.

Q1
A sequence is defined by the recurrence relation $u_{n+1} = 2u_n + 3$ and $u_0 = 1$.
What is the value of u_4?

Q2
A sequence is defined by the recurrence relation $u_{n+1} = \frac{1}{3}u_n + 10$ with $u_2 = -12$.
(a) What is the value of u_3?

(b) Explain why this sequence approaches a limit as $n \to \infty$?

(c) Calculate this limit.

Q3
(a) A sequence is defined by $u_{n+1} = -\frac{1}{2}u_n$ with $u_0 = -16$.
Write down the values of u_1 and u_2.

(b) A second sequence is given by 4, 5, 7, 11, …
It is generated by the recurrence relation $v_{n+1} = pv_n + q$ with $v_1 = 4$. Find the values of p and q.

(c) Either the sequence in (a) or the sequence in (b) has a limit. Calculate this limit.

Q4

A frog and a toad fall to the bottom of a well that is 50 feet deep.
Each day the frog climbs 32 feet then rests overnight.
Overnight it slides down ⅔ of its height above the floor of the well.
The toad climbs 13 feet each day before resting.
Overnight it slides down ¼ of its height above the floor of the well.
Their progress can be modelled by the recurrence relations:

$f_{n+1} = \frac{1}{3}u_n + 32$ with $f_1 = 32$
and
$t_{n+1} = \frac{3}{4}t_n + 13$ with $t_1 = 13$

where f_n and t_n are the heights reached by the frog and the toad at the end of the n^{th} day after falling in.

(a) Calculate, the height of the toad at the end of the second day.

(b) Determine whether or not either of them will eventually escape from the well.

Q5

A sequence is generated by the recurrence relation
$u_{n+1} = mu_n + 6$, where m is a constant.

(a) Given $u_1 = 28$ and $u_2 = 13$, find the value of m.

(b) Explain why this sequence approaches a limit as $n \rightarrow \infty$?

(c) Calculate this limit.

Q6

Sequences can be generated by the
recurrence relations of the form $u_{n+1} = ku_n - 20$, $u_0 = 5$ where $k \in R$

(a) Show that $u_2 = 5k^2 - 20k - 20$.

(b) Determine the range of values of k for which $u_2 < u_0$.