

Please attempt the following sketches in preparation for the online session on 23rd February.

Q1

A sequence is defined by the recurrence relation

$$u_{n+1} = 3u_n - 4 \text{ and } u_0 = -1.$$

What is the value of u_2 ?

Q2

The population of hamsters in a breeding centre increases by 5% each month. At the end of each month the breeder sells 30 hamsters. If u_n represents the hamster population at the beginning of a month, find an expression for u_{n+1} .

Q3

A sequence is defined by the recurrence relation

$$u_{n+1} = au_n + b \text{ and } u_0 = 4.$$

Express u_2 in terms of **a** and **b**.

Q4

A sequence is defined by the recurrence relation

$$u_{n+1} = \frac{1}{3}u_n - 7 \text{ and } u_0 = -2.$$

What is the limit of this sequence as $n \rightarrow \infty$?

Q5

A sequence is defined by the recurrence relation

$$u_{n+1} = 0.6u_n + k \text{ and } u_0 = 3.$$

As $n \rightarrow \infty$, the limit of this sequence is 5.

What is the value of k ?

Q6

A sequence is defined by the recurrence relation

$$u_{n+1} = \frac{2}{3}u_n + 5 \text{ and } u_3 = 9.$$

- Find the value of u_4 .
- Explain why this sequence approaches the limit as $n \rightarrow \infty$
- Calculate the limit of this sequence.

Q7

A recurrence relation is defined by

$$u_{n+1} = pu_n + q, \text{ where } -1 < p < 1 \text{ and } u_0 = 12.$$

If $u_1 = 15$ and $u_2 = 16$, find the values of p and q .

Q8

A sequence is defined by the recurrence relation

$$u_{n+1} = (p - 1)u_n + 3 \text{ and } u_0 = 12.$$

For what values of p does this sequence have a limit?

Q9

A man decides to plant a fast-growing hedge between his house and his neighbour next door. He has been warned that, in any year, the hedge is expected to increase in height by 50cm. Because of this he decides to cut 20% off the height of the hedge at the start of each year.

- If he adopts the “20% pruning policy”, what height will he expect the hedge to reach in the long run?
- His neighbour is concerned that the hedge is growing too fast and asks that the trees be allowed to grow no taller than 2 metres. What is the minimum percentage that the hedge will need to be trimmed each year to meet this condition?

Q10

A geometric sequence is defined by

$$u_{n+1} = -\frac{1}{2}u_n \text{ with } u_0 = -16$$

a) What is the value of u_2 ?

A second sequence is given by 4, 5, 7, 11,

It is generated by the recurrence relation

$$v_{n+1} = pv_n + q \text{ with } v_1 = 4$$

b) What are the values of p and q ?

c) Which sequence has a limit and why?

d) What is the limit of this sequence?